Coherence and decoherence of a polariton condensate

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We formulate a Langevin-Gross-Pitaevski equation for spatially homogeneous systems together with the semiclassical Boltzmann equations for the excited states of polaritons in a microcavity. The gain of the coherent amplitude is due to the polariton-polariton scattering from the excited states to the ground state and has been obtained by an adiabatic elimination the corresponding three-point polarization. The Langevin-Gross-Pitaevski equation contains in addition to the gain the cavity losses as well as the fluctuations from the cavity losses and from the eliminated polarization. In analogy to the semiconductor laser theory the homogeneously broadened linewidth of the condensate amplitude can be evaluated analytically above threshold using the dissipation-fluctuation theorem. A linewidth enhancement is found because of the changes in the dispersive part of the gain function with the number of excited states and of the blueshift of the ground state. The latter mechanism causes well above threshold the homogeneously broadened linewidth to increase again after a remarkably narrow linewidth is reached. This decoherence mechanisms is inherent to all nonequilibrium condensates due to the Gross-Pitaevski nonlinearity and the fluctuations of the condensate population

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I. INTRODUCTION

The exciton polaritons in microcavities have been shown in many experiments to condense above threshold in the ground state of the lower polariton branch. At this point phase coherence of the condensate evolves which can be seen by a Schawlow-Townes decrease in the emission linewidth. But only at slightly higher values of the condensate populations the linewidth is observed to go through a sharp minimum, above which a decoherence sets in manifested by a sharp linewidth increase.[1,2] As seen in a recent investigation[3] these earlier measurements have been dominated by pump noise, using a noise-free diode laser the linewidth decrease much further down to 10 μeV which corresponds to a coherence time of about 150 ps. Already Yamamoto *et al.*[4] in the framework of a Langevin theory and Porras and Tejedor[5] using a reduced density-matrix formulation, recognized that this linewidth increase is due to the blueshift due caused by the polariton-polariton (p-p) interaction in the ground state. However, analytical results for the linewidth have only be given in limiting cases.

In order to study the inherent mechanism for this rerising of the linewidth and the connected decoherence mechanism we consider the homogeneous limit. A linewidth theory for homogeneous broadening needs a description of the dynamics of the condensate amplitude. The theory of laser and of atom lasers provides us with the tools for such studies. We will us a quantum Langevin equation for the coherent condensate amplitude which will allow us to derive an analytical linewidth formula in close analogy with the well-established linewidth enhancement theory of semiconductor lasers.[6,7] The dominant gain mechanism is the p-p scattering which scatters one excited particle from state $\vec{q}$ to the ground state 0 while another one is scattered into a state with higher energy from $\vec{k}-\vec{q}$ to $\vec{k}$. This scattering process couples a three-point variable of the excited states, called the polarization, to the amplitude equation. An adiabatic elimination of the polarization provides the gain, the frequency shift, and fluctuations due to these scattering processes. In the Markov approximation the second moments of the fluctuation operators are determined by the dissipation-fluctuation theory. Because in the microcavity (mc) system the excited states are band states as, e.g., in a semiconductor laser, we use the analogy with the theory of this laser type[6] in the following.

II. DERIVATION OF THE LANGEVIN-GROSS-PITAEVSKI EQUATION

We start our treatment by selecting the interaction which provides the gain of the condensate. The corresponding interaction Hamiltonian is

$$H_{int} = \sum_{\vec{k},\vec{q}} \hbar g_q (b^\dagger_{\vec{k}-\vec{q}} b^\dagger_0 b_{\vec{k}} b_{\vec{q}} + \text{H.c.}). \qquad (1)$$

Here g_q is the p-p interaction, originating from the Coulomb exchange energy between two quantum well excitons, multiplied by the exciton Hopfield coefficients of the four involved polaritons. Both wave numbers k and q are unequal zero. The Heisenberg equation for the ground-state operator $b_0(t)$ together with the losses and connected fluctuations yields

$$\frac{db_0}{dt} = -i\omega_0(\hat{n}_0)b_0 - \gamma_0 b_0 - i\sum_{\vec{k},\vec{q}} g_q b^\dagger_{\vec{k}} b_{\vec{k}-\vec{q}} b_{\vec{q}} + F_0(t), \qquad (2)$$

where γ_0 is the cavity-loss rate and $F_0(t)$ the connected Langevin fluctuation operator. The frequency of the ground state is dependent on the condensate population operator $\hat{n}_0 = b^\dagger_0 b_0$

$$\omega_0(\hat{n}_0) = \omega_0 + g_0\hat{n}_0. \tag{3}$$

The last term in Eq. (3) is due to the self-interaction of the polaritons in the ground state and presents—if averaged—the well-known blueshift. The operator of the polarization

$$P_{k,q} = b^\dagger_{\vec{k}} b_{\vec{k}-\vec{q}} b_{\vec{q}} \tag{4}$$

couples to the coherent amplitude, just as the optical polarization couples to the laser mode. In order to get a closed system of equations for b_0 and the densities of the excited states we eliminate the polarization adiabatically. For this purpose we evaluate its equation of motion

$$\frac{dP_{\vec{k},\vec{q}}}{dt} = \frac{i}{\hbar}[H_{int}, P_{\vec{k},\vec{q}}]. \tag{5}$$

The commutator yields the following operator products:

$$\sum_{\vec{k}',\vec{q}'} g_{q'}(b^\dagger_{\vec{k}'-\vec{q}'} b_{\vec{k}'} b^\dagger_{\vec{q}'} b^\dagger_{\vec{k}} b_{\vec{k}-\vec{q}} b_{\vec{q}} - b^\dagger_{\vec{k}} b_{\vec{k}-\vec{q}} b_{\vec{q}} b^\dagger_{\vec{k}'-\vec{q}'} b_{\vec{k}'} b_{\vec{q}'}) b_0. \tag{6}$$

A straightforward factorization in terms of excited-state densities yields together with the damping $\gamma_{\vec{k},\vec{q}}$ and the fluctuations $F_{\vec{k},\vec{q}}$ of the polarization

$$\frac{dP_{\vec{k},\vec{q}}}{dt} = [i(\omega_{\vec{k}} - \omega_{\vec{k}-\vec{q}} - \omega_{\vec{q}}) - \gamma_{\vec{k},\vec{q}}]P_{\vec{k},\vec{q}} + ig_q \times [n_{\vec{q}} n_{\vec{k}-\vec{q}}(1+n_{\vec{k}}) - n_{\vec{k}}(1+n_{\vec{q}})(1+n_{\vec{k}-\vec{q}})]b_0 + F_{\vec{k},\vec{q}}. \tag{7}$$

Taking the rapid oscillations out of the condensate amplitude

$$b_0(t) = B_0(t)e^{-i\Omega_0 t} \tag{8}$$

one gets the adiabatic solution

$$P_{\vec{k},\vec{q}}(t) = g_q \frac{n_{\vec{q}} n_{\vec{k}-\vec{q}}(1+n_{\vec{k}}) - n_{\vec{k}}(1+n_{\vec{q}})(1+n_{\vec{k}-\vec{q}})}{(\omega_{\vec{k}} - \omega_{\vec{k}-\vec{q}} - \omega_{\vec{q}} + \Omega_0) + i\gamma_{\vec{k},\vec{q}}} b_0 + \int_{-\infty}^{t} dt' F_{\vec{k},\vec{q}}(t') e^{i(\omega_{\vec{k}} - \omega_{\vec{k}-\vec{q}} + \omega_{\vec{q}} + i\gamma_{\vec{k},\vec{q}})(t-t')}. \tag{9}$$

In detail this result can be justified by pulling the slowly varying occupation factors of the excited states out of the time integral on the upper limit t and by performing the time integration over the rapidly varying frequencies. In this sense the numbers of the excited states can still be considered to vary adiabatically with time. Inserting this result back into the Langevin-Gross-Pitaevski equation we find

$$\frac{db_0}{dt} = -i(\omega_0(\hat{n}_0) + G'')b_0 + (G' - \gamma_0)b_0 + F_0(t) - i\sum_{\vec{k},\vec{q}} g_q \int_{-\infty}^{t} dt' F_{\vec{k},\vec{q}}(t') e^{i(\omega_{\vec{k}} - \omega_{\vec{k}-\vec{q}} - \omega_{\vec{q}} + i\gamma_{\vec{k},\vec{q}})(t-t')}, \tag{10}$$

where in the limit of vanishing damping the gain of the condensate $G'(N_x)$ is

$$G' = \sum_{\vec{k},\vec{q}} g_q^2 [n_{\vec{q}} n_{\vec{k}-\vec{q}}(1+n_{\vec{k}}) - n_{\vec{k}}(1+n_{\vec{q}})(1+n_{\vec{k}-\vec{q}})] \times \pi\delta(\omega_{\vec{k}} - \omega_{\vec{k}-\vec{q}} - \omega_{\vec{q}} + \Omega_0) \tag{11}$$

and the imaginary part is

$$G'' = P\sum_{\vec{k},\vec{q}} g_q^2 \frac{n_{\vec{q}} n_{\vec{k}-\vec{q}}(1+n_{\vec{k}}) - n_{\vec{k}}(1+n_{\vec{q}})(1+n_{\vec{k}-\vec{q}})}{\omega_{\vec{k}} - \omega_{\vec{k}-\vec{q}} - \omega_{\vec{q}} + \Omega_0}, \tag{12}$$

where P denotes the principal value. The two expressions of the complex gain have been obtained by applying the Dirac identity. G'' describes the energy shift due to the interaction of the condensate with the excited states. G' and G'' describe real and imaginary parts of a complex gain function. The Eq. (10) can be seen as the nonequilibrium extension of the Gross-Pitaevski equation. Wouters and Carusotto[8] used such a phenomenologically derived equation (without fluctuations) for spatially inhomogeneous systems together with one kinetic equation for the number of excited states in order to calculate the excitation spectrum of a nonequilibrium condensate. Recently, a stochastic extension of these equations in terms of a functional Fokker-Planck equation has been given by Wouters and Savona.[9] A similar formulation mainly concerned with the polarization coherence is due to Read *et al.*[10] As is well known already from laser theory the Fokker-Planck and the Langevin noise theories are equivalent. On the other hand each formulation allows in the treatment of concrete problems specific approximations not easily accessible in the alternative formulation.

The first moments of the fluctuation operators vanish, the second moments are in the Markov limit delta correlated. The dissipation-fluctuation theorem for the order parameter fluctuations are[11]

$$\langle F_0^\dagger(t)F_0(t') + F_0(t)F_0^\dagger(t')\rangle = \delta(t-t')2\gamma_0(2n_{0,th}+1). \tag{13}$$

$n_{0,th}$ is the thermal number of ground-state polaritons, which can be neglected. $\langle FF\rangle$ and $\langle F^\dagger F^\dagger\rangle$ are both zero. For the Langevin equation

$$\frac{dP_{\vec{k},\vec{q}}}{dt} = A_{\vec{k},\vec{q}} + F_{\vec{k},\vec{q}}(t) \tag{14}$$

the polarization fluctuation moments can be calculated from the fluctuation-dissipation theorem[12,13]

$$\langle F_{\vec{k},\vec{q}}(t)F^\dagger_{\vec{k}',\vec{q}'}(t')\rangle = \delta(t-t')\delta_{\vec{k},\vec{k}'}\delta_{\vec{q},\vec{q}'}\langle F_{\vec{k},\vec{q}}F^\dagger_{\vec{k},\vec{q}}\rangle, \tag{15}$$

where we need, in particular,

$$\langle F_{\vec{k},\vec{q}}F^\dagger_{\vec{k},\vec{q}}\rangle + \langle F^\dagger_{\vec{k},\vec{q}}F_{\vec{k},\vec{q}}\rangle = 2\gamma_{\vec{k},\vec{q}}[n_{\vec{q}} n_{\vec{k}-\vec{q}}(1+n_{\vec{k}}) + n_{\vec{k}} \times (1+n_{\vec{q}})(1+n_{\vec{k}-\vec{q}})]. \tag{16}$$

The polarization fluctuations are determined by the sum of the two population factors whose difference determines the gain.

III. LINEARIZATION OF PHASE AND AMPLITUDE EQUATIONS

With this formalism we can now calculate the linewidth, given by the second moment of the phase fluctuation and the density fluctuations above and below the condensate threshold. The average over an exponential of the fluctuating phase $\phi(t)$ yields

$$\langle e^{i[\phi(t)-\phi(0)]}\rangle \simeq e^{-\kappa t} \tag{17}$$

with the linewidth $\kappa = \frac{1}{2}\frac{\langle[\phi(t)-\phi(0)]^2\rangle}{t}$, which has the form a phase-diffusion coefficient. For simplicity we consider stationary situations of the Langevin-Gross-Pitaevsky equation with fluctuations around a stationary mean value. Above threshold we use the semiclassical decomposition of the fluctuating condensate amplitude

$$b_0(t) = [r_0 + \rho(t)]e^{[i\Omega_0 + i\phi(t)]}, \tag{18}$$

where the fluctuating part of the amplitude can be assumed to be much smaller than the coherent part, i.e., $\langle \rho(t)^2\rangle \ll r_0^2 = N_0$.

The complex gain function $G(N_x)$ depends on the total number of particles N_x in the excited states, which in local equilibrium can be related by its kinetics equation to N_0, i.e., $G[N_x(N_0)]$, as will be discussed below. With this approach we assume that the kinetics of the excited states follows the order parameter instantaneously. This bypass of the excited states kinetics allows to get simple analytical expressions for the linewidth.

With this ansatz we get from the Langevin-Gross-Pitaevski equation first two identities from the mean part

$$\Omega_0 = \omega_0 + g_0 N_0 + G_0''(N_x) \quad \text{and} \quad G_0'(N_x) = \gamma_0. \tag{19}$$

The frequency shifts are due to the particle-particle interaction and the dispersive effect of the scattering from the excitation continuum into the ground state. The second relation shows that above threshold the mean saturated gain equals the losses for the condensate.

Using the above-mentioned expansion the equations for the fluctuating phase and amplitude are

$$r_0\dot{\phi} = -\left(g_0 + \frac{\partial G''}{\partial N_0}\right)2N_0\rho + \mathrm{Im}[\tilde{F}_0(t)e^{i\Omega_0 t}], \tag{20}$$

$$\dot{\rho} = +\frac{\partial G'}{\partial N_0}2N_0\rho + \mathrm{Re}[\tilde{F}_0(t)e^{i\Omega_0 t}], \tag{21}$$

where $\tilde{F}_0$ is given by

$$\tilde{F}_0(t) = F_0(t) - i\sum_{k,q} g_q \int_0^t dt_1 F_{k,q}(t_1)e^{i(\Omega_k - \Omega_{k-q} - \Omega_q + i\gamma_{k,q})(t-t_1)}. \tag{22}$$

Here we generalized the perturbational result by replacing the bare frequencies by the renormalized ones $\omega_k \to \Omega_k$ which are shifted in the same way as the ground state in order to avoid an artificial gap. The same replacements are made in the gain formula (11) and in the corresponding shift G''. In the above given Eqs. (20) and (21) $\frac{\partial G}{\partial N_0}$ has to be expressed by changes in G with N_x. Note that with fluctuations the gain is not constant, as the mean gain result in Eq. (19) suggests, but approaches the loss rate asymptotically, so that the derivative of the complex gain with either N_0 or N_x are finite.

The mean summed stationary rate equation for the excited states provides the functional relation for the changes in N_x with N_0. In particular the derivative of the complex gain function G with respect to N_0 which appears in the linearized equations is

$$\frac{\partial G}{\partial N_0} = \frac{\partial G}{\partial N_x}\frac{dN_x}{dN_0}. \tag{23}$$

The mean stationary rate equation for total number of excited particles is

$$\dot{N}_x = P - 2\bar{\gamma}N_x - 2G' N_0 = 0, \tag{24}$$

where P is the total noise-free pump rate and $\bar{\gamma}$ is the mean loss rate, defined by

$$\bar{\gamma} = \frac{\sum_k \gamma_k n_k}{\sum_k n_k} = \frac{\sum_k u_k^2 \gamma_0 n_k}{N_x}. \tag{25}$$

Here u_k is the photon Hopfield coefficient of the mc polaritons. Note that the scattering within the excited states drops out due to detailed balance, once a local equilibrium is established. Both in experiment[14] and in the Boltzmann kinetic studies[15] it has been shown that even with picosecond pulse excitation such a local equilibrium is reached about 40 ps after the pulse. Under these conditions a reduction in the Boltzmann equation for the excited states to the simple-rate equation for all excited states is possible. From Eq. (24) we find

$$-\bar{\gamma}dN_x - \frac{\partial G'}{\partial N_x}N_0 dN_x - G' dN_0 = 0, \tag{26}$$

which yields

$$\frac{dN_x}{dN_0} = -\frac{G'}{\bar{\gamma}}\frac{1}{1+\frac{N_0}{N_s}} = = -\frac{\gamma_0}{\bar{\gamma}}\frac{1}{1+\frac{N_0}{N_s}}, \tag{27}$$

where we inserted the mean equation result in Eq. (19) for the saturated gain. The gain saturation number is given by

$$N_s^{-1} = \frac{1}{\bar{\gamma}}\frac{\partial G'}{\partial N_x}. \tag{28}$$

This gain saturation should not be confused with the often quoted saturation density which indicates the breakdown of the boson model for quantum well excitons. Finally the derivative of the complex gain with respect to N_0 is

$$\frac{\partial G}{\partial N_0} = -\frac{\partial G}{\partial N_x}\frac{\gamma_0}{\bar{\gamma}}\frac{1}{1+\frac{N_0}{N_s}}. \tag{29}$$

For the real part of the gain the result can be simplified further using the definition of N_s

$$\frac{\partial G'}{\partial N_0} = -\gamma_0\frac{\frac{1}{N_s}}{1+\frac{N_0}{N_s}}. \tag{30}$$

These results have to be inserted in the linearized Eqs. (20) and (21) for the fluctuating phase and amplitude. We emphasize again, the advantage of this approach to shortcut the fluctuations of the excited-states population by expressing the changes in the gain with the number of excited particles by the changes in the gain with N_0 is that the linearized noise equations are simpler and allow analytical results for the linewidth and the second-order correlation function. On the other hand, the treatment of both condensate fluctuations and independent fluctuations of the excited states is certainly the more complete approach.

The integral of the amplitude equation is

$$\rho(t) = \int_0^t dt_1\, \mathrm{Re}[\tilde{F}_0(t_1)e^{i\Omega_0 t_1}]e^{-2\gamma_0(N_0/N_s)/1+(N_0/N_s)t_1}. \tag{31}$$

Inserting this result into the phase equation we find

$$\begin{aligned} r_0[\phi(t)-\phi(0)] = -\int_0^t dt_2\Bigg\{&2N_0\left(g_0+\frac{\partial G''}{\partial N_0}\right)\int_0^{t_2}dt_1 \\ &\times[\mathrm{Re}\,\tilde{F}_0(t_1)e^{i\Omega_0 t_1}]e^{\partial G'/\partial N_0 2N_0 t_1} \\ &+\mathrm{Im}[\tilde{F}_0(t_2)e^{i\Omega_0 t_2}]\Bigg\} \end{aligned} \tag{32}$$

or

$$\begin{aligned} &\langle[\phi(t)-\phi(0)]^2\rangle \\ &= \frac{1}{N_0}\int_0^t dt_2\int_0^t dt_4 \\ &\times\Bigg\langle\Bigg(\Bigg\{\int_0^{t_2}dt_1 2N_0\left(g_0+\frac{\partial G''}{\partial N_0}\right) \\ &\times\mathrm{Re}[\tilde{\tilde{F}}_0(t_1)e^{i\Omega_0 t_1}]e^{\partial G'/\partial N_0 2N_0 t_1}+\mathrm{Im}[\tilde{F}_0(t_2)e^{i\Omega_0 t_2}]\Bigg\} \\ &\times\Bigg\{\int_0^{t_4}dt_3 2N_0\left(g_0+\frac{\partial G''}{\partial N_0}\right)\mathrm{Re}[\tilde{F}_0(t_3) \\ &\times e^{i\Omega_0 t_3}]e^{\partial G'/\partial N_0 2N_0 t_3}+\mathrm{Im}[\tilde{F}_0(t_4)e^{i\Omega_0 t_4}]\Bigg\}\Bigg)\Bigg\rangle. \end{aligned} \tag{33}$$

In all these expressions the derivative of the complex gain function with respect to N_0 has to be expressed by Eq. (29) in terms of derivatives with respect to N_x.

IV. LINEWIDTH AND SECOND-ORDER CORRELATION FUNCTION

Considering first only the order parameter fluctuations, we use

$$\begin{pmatrix}\mathrm{Re}\\ i\,\mathrm{Im}\end{pmatrix}F_0e^{i\Omega_0 t} = \frac{1}{2}(F_0e^{i\Omega_0 t}\pm F_0^\dagger e^{-i\Omega_0 t}) \tag{34}$$

and the Markov dissipation-fluctuation theorem [Eq. (13)]. For $\gamma_0 t \gg 1$ and for

$$\gamma_0 t\frac{\frac{N_0}{N_s}}{1+\frac{N_0}{N_s}} \gg 1 \tag{35}$$

one finds the linewidth contribution

$$\kappa = \frac{\gamma_0}{2N_0}(2n_{th}+1)\left\{1+\left[\frac{g_0\left(1+\frac{N_0}{N_s}\right)\frac{\bar{\gamma}}{\gamma_0}-\frac{\partial G''}{\partial N_x}}{\frac{\partial G'}{\partial N_x}}\right]^2\right\}. \tag{36}$$

Here $n_{th}=\frac{1}{e^{\hbar\Omega_0\beta}-1}$ is the negligibly small thermal polariton number in the ground state, the 1 represents the contribution of the vacuum fluctuations. If we take additionally the polarization fluctuation in $\tilde{F}_0$ into account the final linewidth result becomes

$$\kappa = \frac{\gamma_0}{N_0}(n_{th}+n_{sp})\left\{1+\left[\frac{g_0\left(1+\frac{N_0}{N_s}\right)\frac{\bar{\gamma}}{\gamma_0}-\frac{\partial G''}{\partial N_x}}{\frac{\partial G'}{\partial N_x}}\right]^2\right\}. \tag{37}$$

The appearance of the number of spontaneously emitted bosons into the ground state can be understood as follows. We express the gain function [see Eq. (11)] as $G'=R_{x\to 0}-R_{0\to x}$, i.e., as the difference between the transition rates from the excited states to the ground state and that of the reverse processes. We find from the polarization fluctuations with Eq. (16) plus the vacuum fluctuations the contribution $\gamma_0(2n_{th}+1)+R_{x\to 0}+R_{0\to x}$. Adding and subtracting the first rate, we find $2\gamma_0 n_{th}+2R_{x\to 0}-G'+\gamma_0=2\gamma_0(n_{th}+n_{sp})$, where we made use of Eq. (19). The number of spontaneously emitted ground-state polaritons is determined by the simple-rate equation

$$2\gamma_0 n_{sp} = 2R_{x\to 0}. \tag{38}$$

For practical purpose $n_{sp}\gg n_{th}$.

Without the square enhancement term, the linewidth Eq. (37) has the form of a Schawlow-Townes linewidth which decreases as $1/N_0$ above threshold. The square term has the form of a linewidth enhancement factor and reduces for $g_0=0$ to the well-known enhancement factor

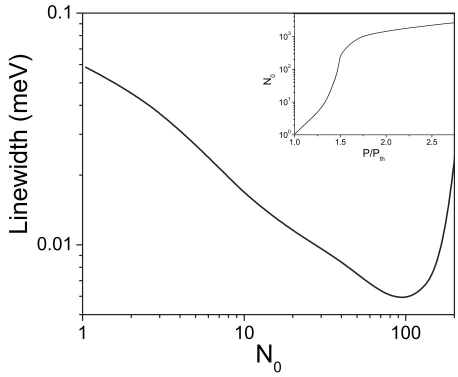

FIG. 1. Calculated linewidth for GaAs-based mc's above threshold versus condensate population N_0. Inset: condensate population N_0 versus normalized pump power.

$$\alpha^2 = \left(\frac{\partial G_x''/\partial N_x}{\partial G_x'/\partial N_x}\right)^2$$

from the semiconductor laser theory.[6,7] Because the derivatives of real and imaginary gains enter in the linewidth with the saturation denominator in Eq. (29), the term $(1+N_0/N_s)$ finally becomes in Eq. (37) a prefactor of the self-interaction matrix element g_0. Note the opposite signs of this blueshift part and the part due to the changes in the imaginary gain function $G''(N_x)$ with the number of excited polaritons N_x (because $\partial G''/\partial N_x > 0$). Directly above threshold the latter part dominates, so that a Schawlow-Townes law $\frac{1}{N_0}$ can be observed with an α^2-enhanced linewidth due to the interaction with the excited states. Eventually the square term in N_0 dominates over the $1/N_0$ prefactor. The linewidth calculated with Eq. (37) using the solutions of the Boltzmann kinetics is shown in Fig. 1 for GaAs-based mc's. It is seen that a remarkably small linewidth is reached before at higher pump values the decoherence mechanism dominates. For this calculation the complex gain function and the derivatives $\partial G'/\partial N_x$ and $\partial G''/\partial N_x$ had to be evaluated using the Boltzmann kinetics for the excited-states population n_k. The threefold integrals over the involved wave numbers k, q and the angle between the corresponding wave vectors are not easily evaluated with the required accuracy and depend, e.g., sensitively on the assumed finite-polarization damping $\gamma_{k,q}$ [see Eq. (9)] which was assumed to be 0.2 meV for the results shown in Fig. 2. On the other hand the position and depth of the minimum are very susceptible to these values, thus an accurate prediction of the value of the minimal linewidth and the corresponding pump power is difficult. The calculated derivatives of the gain with respect to the number of excited particles are shown in Fig. 2. The resulting gain saturation number N_s which according to Eq. (28) is proportional to the inverse of $\partial G'/\partial N_x$ varies in the shown pump power range between 2×10^3 and 4×10^3.

As mentioned in the introduction, under quasistationary conditions a Gaussian decay with very long coherence times of up to 150 ps has been observed in CdTe-based mc's using noise-free pumping.[3] This decay time corresponds to a linewidth of about 10 μeV, while Porras and Tejedor[5] estimated a minimal linewidth of 1 μeV. In our simplified model in

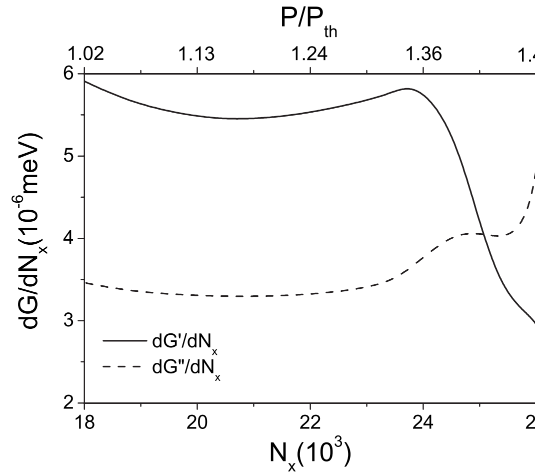

FIG. 2. Calculated derivatives of the complex gain as a function of the excited-states population N_x. On the top of the figure the corresponding values of the normalized pump power are given.

which the noise of the excited-states population is bypassed, we get for GaAs mc's an exponential decay with a minimal linewidth of about 60 μeV at a condensate population $N_0 \simeq 100$. This minimum linewidth condensate number is of the same order as that observed in pump-noise-free experiments.[3]

Instead of a detailed kinetic analysis, the analytical linewidth formula (37) can also be used to characterize with a few fitting parameters such as n_{sp}, N_s, and real and imaginary parts of $\partial G/\partial N_x$ a measured linewidth around its minimal value. It is important to state that the nonlinear Gross-Pitaevski self-interaction term together with the density fluctuations provides for all nonequilibrium condensates a decoherence mechanism which increases the phase correlations and thus the linewidth.

Similarly the second-order correlation function $g_2(\tau,t)$ can be evaluated from the amplitude fluctuations

$$\begin{aligned} g_2(\tau,t) &= \frac{\langle b_0^\dagger(t) b^\dagger(t+\tau)_0 b(t+\tau)_0 b(t)_0\rangle}{\langle b_0^\dagger(t) b(t)_0\rangle\langle b^\dagger(t+\tau)_0 b(t+\tau)_0\rangle} \\ &= 1 + \frac{4}{N_0}\langle\rho(t+\tau)\rho(t)\rangle \\ &= 1 + \frac{4}{N_0}\langle\rho^2(t)\rangle e^{-2\gamma_0 (N_0/N_s)/1+(N_0/N_s)\tau}. \end{aligned} \tag{39}$$

The second variance of the density fluctuations is

$$\langle\rho^2(t)\rangle = \frac{1}{4}\left(1 + \frac{N_s}{N_0}\right)(n_{th} + n_{sp}). \tag{40}$$

Note that the variance saturates at $\frac{1}{4}(n_{th}+n_{sp})$ for $N_0 \gg N_s$, while $g_2(\tau=0)$ still approaches the coherent limit of 1 like N_0^{-1} for very large values of N_0. This asymptotic behavior of the second-order correlation function is still a not completely settled problem (see Refs. 16 and 17). The influence of the spin degree of freedom on first- and second-order coherences has been analyzed in Ref. 18.

In conclusion, an analytical formula for the homogeneous linewidth of a microcavity polariton condensate has been derived from a Langevin-Gross-Pitaevsky equation. The usual Schawlow-Townes linewidth is enhanced by dispersive gain variations known from semiconductor lasers and by the combined effect of the density-dependent blueshift, the gain saturation and the density fluctuations. The nonlinear term of the Gross-Pitaevski equation which is crucial for the coherent properties of interacting equilibrium condensates, rather becomes at higher pump levels a source of decoherence for nonequilibrium condensates because of the fluctuations in the ground-state population. Together with the solutions of the Boltzmann equation for the excited states and the ground state, the derived formula allows a complete evaluation of the linewidth.

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